Estimating Production Functions with Expectations Data

Agnes Norris Keiller [1]   Áureo de Paula [2]
John Van Reenen [3]

[1]LSE
[2]UCL, IFS and CeMMAP
[3]LSE and NBER

Spring NBER Productivity meetings, April 2026

Outline

Outline

Motivation

Methodology: Model, Identification and Estimation

Monte Carlo Simulation

Application

Discussion

Motivation

- Production functions integral to economics

$$y_{it} = f(k_{it}, l_{it}; \theta) + u_{it}$$

  - y_{it} = log(output) of firm i at time t
  - l_{it} = log(labor), k_{it} = log(capital), u_{it} = error term

Motivation

- Production functions integral to economics

$$y_{it} = f(k_{it}, l_{it}; \theta) + u_{it}$$

  - y_{it} = log(output) of firm i at time t
  - l_{it} = log(labor), k_{it} = log(capital), u_{it} = error term
- Used in wide variety of contexts.
  - Productivity dynamics
  - Are factors paid marginal products (e.g., wage mark-downs)?
  - Are prices above marginal costs (mark-ups)?
  - What are the drivers of productivity?

Motivation

- Production functions integral to economics

$$y_{it} = f(k_{it}, l_{it}; \theta) + u_{it}$$

  - y_{it} = log(output) of firm i at time t
  - l_{it} = log(labor), k_{it} = log(capital), u_{it} = error term
- Used in wide variety of contexts.
  - Productivity dynamics
  - Are factors paid marginal products (e.g., wage mark-downs)?
  - Are prices above marginal costs (mark-ups)?
  - What are the drivers of productivity?
- Surveys increasingly elicit firm-level expectation data on future inputs & outputs (e.g., US MOPS. Tab 3 lists 9 surveys in China, Japan, DE...)

Motivation

- Production functions integral to economics

$$y_{it} = f(k_{it}, l_{it}; \theta) + u_{it}$$

  - y_{it} = log(output) of firm i at time t
  - l_{it} = log(labor), k_{it} = log(capital), u_{it} = error term
- Used in wide variety of contexts.
  - Productivity dynamics
  - Are factors paid marginal products (e.g., wage mark-downs)?
  - Are prices above marginal costs (mark-ups)?
  - What are the drivers of productivity?
- Surveys increasingly elicit firm-level expectation data on future inputs & outputs (e.g., US MOPS. Tab 3 lists 9 surveys in China, Japan, DE...)
- Such data enables us to develop a new production function estimator npr

How to estimate the production function?

- Marschak and Andrews (1944) simultaneity problem. Solutions?

How to estimate the production function?

- Marschak and Andrews (1944) simultaneity problem. Solutions?

1. Residual approaches using first order conditions e.g., Solow (1957); Hall (1988). But assumes many things we'd like to test

How to estimate the production function?

- Marschak and Andrews (1944) simultaneity problem. Solutions?

1. Residual approaches using first order conditions e.g., Solow (1957); Hall (1988). But assumes many things we'd like to test
2. Fixed effect estimator (Hoch, 1955; Mundlak, 1961) usually generates implausible estimates on capital (Griliches & Mairesse, 1998)

How to estimate the production function?

- Marschak and Andrews (1944) simultaneity problem. Solutions?

1. Residual approaches using first order conditions e.g., Solow (1957); Hall (1988). But assumes many things we'd like to test
2. Fixed effect estimator (Hoch, 1955; Mundlak, 1961) usually generates implausible estimates on capital (Griliches & Mairesse, 1998)
3. "External" Instruments like factor input prices (e.g., de Roux et al., 2024)
   - Hard as requires exogenous firm-specific variation in factor prices over time

How to estimate the production function?

- Marschak and Andrews (1944) simultaneity problem. Solutions?

1. Residual approaches using first order conditions e.g., Solow (1957); Hall (1988). But assumes many things we'd like to test
2. Fixed effect estimator (Hoch, 1955; Mundlak, 1961) usually generates implausible estimates on capital (Griliches & Mairesse, 1998)
3. "External" Instruments like factor input prices (e.g., de Roux et al., 2024)
   - Hard as requires exogenous firm-specific variation in factor prices over time
4. "Internal" Instruments
   - Dynamic panel IV: e.g., Blundell & Bond (2000)
   - Proxy methods (control functions): e.g., Olley and Pakes (1996, OP); Levinsohn & Petrin (2003, LP); Ackerberg, Caves & Fraser, (2015, ACF)

How to estimate the production function?

- Marschak and Andrews (1944) simultaneity problem. Solutions?

1. Residual approaches using first order conditions e.g., Solow (1957); Hall (1988). But assumes many things we'd like to test
2. Fixed effect estimator (Hoch, 1955; Mundlak, 1961) usually generates implausible estimates on capital (Griliches & Mairesse, 1998)
3. "External" Instruments like factor input prices (e.g., de Roux et al., 2024)
   - Hard as requires exogenous firm-specific variation in factor prices over time
4. "Internal" Instruments
   - Dynamic panel IV: e.g., Blundell & Bond (2000)
   - Proxy methods (control functions): e.g., Olley and Pakes (1996, OP); Levinsohn & Petrin (2003, LP); Ackerberg, Caves & Fraser, (2015, ACF)
5. NPR brings in exploit **new data** on firm expectations of output and inputs [Card's Maxim]
   - These expectations reveal what a firm *believes* it's productivity will be next period, thus helping reveal what unobserved productivity is *this* period
   - Hence, use functions of observed expectations to get unbiased estimates of production function

Preview

1. **Theory**
   - Expectations data means we do not need to make strong assumptions over firms' input decision rules required by proxy estimators

Preview

1. **Theory**
   - Expectations data means we do not need to make strong assumptions over firms' input decision rules required by proxy estimators
2. **Monte Carlo simulations**
   - Our proposed estimator is robust to optimization error in inputs, while other methods are not (invertibility issue)

Preview

1. **Theory**
   - Expectations data means we do not need to make strong assumptions over firms' input decision rules required by proxy estimators
2. **Monte Carlo simulations**
   - Our proposed estimator is robust to optimization error in inputs, while other methods are not (invertibility issue)
3. **Application**
   - UK firm-level admin panel data: 2 waves of Management & Expectations Survey (MES) + annual ABI
   - NPR particularly useful when choosing inputs optimally hard (e.g., restaurants around Brexit/Covid period).
   - NPR TFP estimates produces more sensible dynamic moments between initial TFP and subsequent growth

NPR

- **Advantages of NPR**
  - Relax strong assumptions on optimization of factor inputs common to all proxy methods (needs strict monotonicity of inputs wrt productivity to allow invertibility)
    - So NPR better under oligopoly; unmeasured input prices,
    - And when hard to make optimal decisions - an “estimator for turbulent times”.
  - Can estimate NPR with a single cross section of data.
    - Need $\geqslant 2$ periods for proxy methods and $\geqslant 3$ periods for dynamic panel estimation
    - recommend adding expectations questions to standard surveys (e.g. World Bank Enterprise Survey) to help deal with unobserved heterogeneity

NPR

- **Advantages of NPR**
  - Relax strong assumptions on optimization of factor inputs common to all proxy methods (needs strict monotonicity of inputs wrt productivity to allow invertibility)
    - So NPR better under oligopoly; unmeasured input prices,
    - And when hard to make optimal decisions - an "estimator for turbulent times".
  - Can estimate NPR with a single cross section of data.
    - Need $\geqslant 2$ periods for proxy methods and $\geqslant 3$ periods for dynamic panel estimation
    - recommend adding expectations questions to standard surveys (e.g. World Bank Enterprise Survey) to help deal with unobserved heterogeneity

- **Disadvantages of NPR**
  - Require explicit firm-level expectations data
  - Some (testable) assumptions over this data (biases, measurement error, etc.)

Some Existing Literature

- **History**: Von Thuenen (1820s); Tolley et al. (1924); Cobb & Douglas (1928); de Loecker & Syverson (2021)
- **Proxy Methods**: Olley & Pakes (1996); Levinsohn & Petrin (2003); Ackerberg, Caves & Fraser (2015)
- **Dynamic Panel Data Methods**: Bond & Soderbom (2005); Blundell & Bond (2000); Anderson & Hsiao (1981); Arellano & Bond (1991)
- **Mark-Ups**: de Loecker and Warzynski (2012); Ackerberg and de Loecker (2025); de Ridder et al. (2024)
- **Extensions**: Gandhi et al., (2020); Orr (2022); Doraszelski-Jaumandreu (2013); Valmari (2023); Demirer (2022); Doraszelski & Jaumandreu (2018); Raval (2023)
- **Expectations**: Dominitz & Manski (1997); Altig et al., (2022); Bloom et al., (2025); Gennaioli et al., (2016); Arellano et al. (2024); Born et al., (2022)

Outline

Motivation

Methodology: Model, Identification and Estimation

Monte Carlo Simulation

Application

Discussion

The object of interest

- Consider a general production function of the following form

$$y_{it} = f(k_{it}, l_{it}; \beta) + \omega_{it} + \epsilon_{it}$$

- $\omega_{it} \equiv$ idiosyncratic productivity known by the firm when deciding period t input and investment
- ϵ_{it} unanticipated mean-zero disturbances
  - measurement error (or productivity shocks unknown by the firm when making period t decisions)

Dynamics

- ω_{it} follows a Markov process

$$\omega_{it} = \mathbb{E}[\omega_{it}|\omega_{it-1}] + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$$

Dynamics

- ω_{it} follows a Markov process

$$\omega_{it} = \mathbb{E}[\omega_{it}|\omega_{it-1}] + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$$

- Capital evolves according to

$$K_{it} = (1 - \delta)K_{it-1} + I_{it-1}$$

where δ = the depreciation rate; I_{it-1} = investment

Proxy Methods

- OP\LP\ACF leverage firm decisions (input demand) and their monotonicity in ω (conditional on observables):

Proxy Methods

- OP\LP\ACF leverage firm decisions (input demand) and their monotonicity in ω (conditional on observables):
  - OP: firms' investment policy $\rightarrow \omega = \Phi^{OP}(i_{it}, k_{it})$
  - LP: firms' material input policy $\rightarrow \omega = \Phi^{LP}(m_{it}, k_{it})$
  - ACF: firms' material input policy $\rightarrow \omega = \Phi^{ACF}(m_{it}, l_{it}, k_{it})$
  - Invertibility of input demand is typically justified by a model of optimal firm decisions

Proxy Methods

- OP\LP\ACF leverage firm decisions (input demand) and their monotonicity in ω (conditional on observables):
  - OP: firms' investment policy $\rightarrow \omega = \Phi^{OP}(i_{it}, k_{it})$
  - LP: firms' material input policy $\rightarrow \omega = \Phi^{LP}(m_{it}, k_{it})$
  - ACF: firms' material input policy $\rightarrow \omega = \Phi^{ACF}(m_{it}, l_{it}, k_{it})$
  - Invertibility of input demand is typically justified by a model of optimal firm decisions
- In NPR, we assume nothing about the invertibility of firms' decisions.

Expectations

- Firms form expectations about $t+1$ production and inputs at the end of t conditional on information set $\Omega_{it} \supset \{K_{it}, L_{it}, K_{it+1}, \omega_{it}\}$

More

Expectations

- Firms form expectations about $t+1$ production and inputs at the end of t conditional on information set $\Omega_{it} \supset \{K_{it}, L_{it}, K_{it+1}, \omega_{it}\}$
- Firm's expectations, $\mathbb{E}_{it}$ are **data.** Don't need to be rational (see App A)

More

Expectations

- Firms form expectations about $t+1$ production and inputs at the end of t conditional on information set $\Omega_{it} \supset \{K_{it}, L_{it}, K_{it+1}, \omega_{it}\}$
- Firm's expectations, $\mathbb{E}_{it}$ are **data.** Don't need to be rational (see App A)

- If firms' expectations align with the true production technology

$$\begin{aligned} \mathbb{E}_{it}[y_{it+1}|\Omega_{it}] &= \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \\ &\quad + \mathbb{E}_{it}[\omega_{it+1}|\Omega_{it}] + \mathbb{E}_{it}[\epsilon_{it+1}|\Omega_{it}] \\ &= \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) + g(\omega_{it}) \end{aligned} \tag{1}$$

  - $F_{it}(l_{it+1}) \equiv$ firm i's subjective prob distribution over next-period labour input
  - $\mathbb{E}_{it}[\xi_{it+1}|\Omega_{it}] = \mathbb{E}_{it}[\epsilon_{it+1}|\Omega_{it}] = 0$

More

Recovering ω_{it}

- Rearranging equation (1) for $g(\omega_{it})$ gives:

$$g(\omega_{it}) = \mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \tag{2}$$

Recovering ω_{it}

- Rearranging equation (1) for $g(\omega_{it})$ gives:

$$g(\omega_{it}) = \mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \tag{2}$$

- Monotonicity assumption means RHS of equation (2) is strictly increasing in ω_{it}

$$\begin{aligned} \omega_{it} &= g^{-1}\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1})\right) \\ &\equiv \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1})\right) \end{aligned}$$

Identification

- Combining

$$\omega_{it} = \Psi \left(\mathbb{E}_{it}[y_{it+1} | \Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1}) \right)$$

with

$$y_{it} = f(k_{it}, l_{it}; \beta) + \omega_{it} + \epsilon_{it}$$

Identification

- Combining

$$\omega_{it} = \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1})\right)$$

with

$$y_{it} = f(k_{it}, l_{it}; \beta) + \omega_{it} + \epsilon_{it}$$

$\Rightarrow$ a moment condition we can use to recover β

$$\begin{aligned} 0 &= \mathbb{E}[\epsilon_{it}|\Omega_{it}] = \mathbb{E}[y_{it} - f(k_{it}, l_{it}; \beta) - \omega_{it}|\Omega_{it}] \\ &= \mathbb{E}\left[y_{it} - f(k_{it}, l_{it}; \beta) - \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \int f(k_{it+1}, l_{it+1}; \beta) dF_{it}(l_{it+1})\right)|\Omega_{it}\right] \end{aligned}$$

Example

Cobb-Douglas production and $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ implies

$$\begin{aligned} y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \\ &= \frac{\rho - 1}{\rho}\beta_0 + \beta_k k_{it} + \beta_l l_{it} + \frac{1}{\rho}\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \frac{\beta_k}{\rho}k_{it+1} - \frac{\beta_l}{\rho}\mathbb{E}_{it}[l_{it+1}|\Omega_{it}] + e_{it} \end{aligned}$$

Example

Cobb-Douglas production and $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ implies

$$\begin{aligned} y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \\ &= \frac{\rho - 1}{\rho}\beta_0 + \beta_k k_{it} + \beta_l l_{it} + \frac{1}{\rho}\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \frac{\beta_k}{\rho} k_{it+1} - \frac{\beta_l}{\rho}\mathbb{E}_{it}[l_{it+1}|\Omega_{it}] + e_{it} \end{aligned}$$

- If measurements on the variables in red are available, we can identify and consistently estimate all the parameters!

Example

Cobb-Douglas production and $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ implies

$$\begin{aligned} y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \\ &= \frac{\rho - 1}{\rho}\beta_0 + \beta_k k_{it} + \beta_l l_{it} + \frac{1}{\rho}\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \frac{\beta_k}{\rho}k_{it+1} - \frac{\beta_l}{\rho}\mathbb{E}_{it}[l_{it+1}|\Omega_{it}] + e_{it} \end{aligned}$$

- If measurements on the variables in red are available, we can identify and consistently estimate all the parameters!
- For a linear AR process and under rational expectations: moment conditions = Blundell and Bond (2000) ($\Rightarrow$ see Bond et al., 2026, AER P&P). Details

Example

Cobb-Douglas production and $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ implies

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it}$$
$$= \frac{\rho - 1}{\rho}\beta_0 + \beta_k k_{it} + \beta_l l_{it} + \frac{1}{\rho}\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \frac{\beta_k}{\rho}k_{it+1} - \frac{\beta_l}{\rho}\mathbb{E}_{it}[l_{it+1}|\Omega_{it}] + e_{it}$$

- If measurements on the variables in red are available, we can identify and consistently estimate all the parameters!
- For a linear AR process and under rational expectations: moment conditions = Blundell and Bond (2000) (⇒ see Bond et al., 2026, AER P&P). Details
- ...but how does one get measurements on $\mathbb{E}_{it}[y_{it+1}|\Omega_{it}]$ and $\mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$?

Yes, Virginia, there is a. . . Management and Expectations Survey!

Looking ahead to the 2021 calendar year, what is the approximate turnover you would anticipate for this business in the following scenarios?

Lowest turnover

Report to the nearest £1,000. For example, £1,357,689 would be reported as £1,358,000

£ 2,800,000

Low turnover

£ 4,200,000

Medium turnover

£ 5,000,000

High turnover

£ 6,300,000

Highest turnover

£ 7,500,000

For the approximate turnover values you have just given for 2021, how likely do you think each scenario is to occur?

Your answers should add up to 100%

Likelihood of lowest turnover occurring

5 %

Likelihood of low turnover occurring

10 %

Likelihood of medium turnover occurring

60 %

Likelihood of high turnover occurring

20 %

Likelihood of highest turnover occurring

5 %

Administrative Data

- **ABI/ABS**: Representative sample of all private sector industries. Panel data on standard production variables (like US ASM, but covers non-manufacturing)
- **MES 2017**: Expectations on sales revenue (turnover), employment, capital expenditure, intermediates expenditure
- **MES 2020**: Expectations on sales & employment

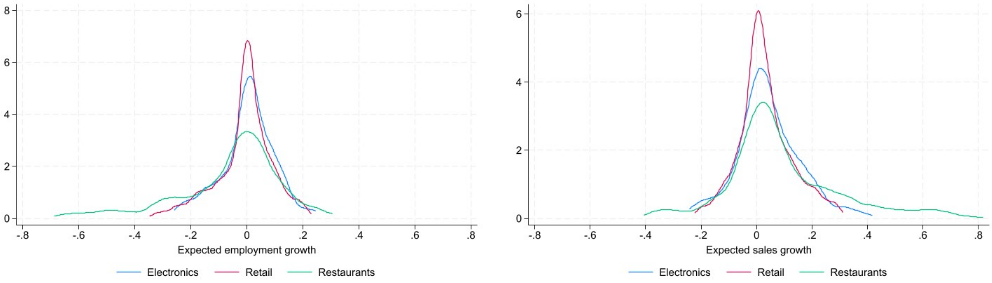
Administrative Data

- **ABI/ABS**: Representative sample of all private sector industries. Panel data on standard production variables (like US ASM, but covers non-manufacturing)
- **MES 2017**: Expectations on sales revenue (turnover), employment, capital expenditure, intermediates expenditure
- **MES 2020**: Expectations on sales & employment
- To get what we need
  - Convert scenario responses into 5 points on a CDF (or 1-CDF)
  - Estimate the parameters of a lognormal distribution to fit these points via least-squares minimization (cf. Dominitz & Manski, 1997)

Predictions and Outcomes

- Use match between expectations and future outcomes (Table 7)
  - On average firms reasonably good at forecasting future sales & employment (see Bloom et al., 2026)
  - Forecasts not significantly different from outcomes (e.g., employment forecast error between -4% and +2% across the industries we focus on)
- But much heterogeneity across firms, especially for restaurants Table 7
- Focusing just on growth predictions .....

Restaurants more uncertain about growth than other industries we focus on (Fig. 3)



Note: SD (obs) of expected log employment growth are 0.09 (462) for Restaurants, 0.05 (2,084) for Retail and 0.04 (472) for Electronics.

Estimation

Under a more general law of motion, Cobb-Douglas production implies

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]\right) + e_{it}$$

Estimation

Under a more general law of motion, Cobb-Douglas production implies

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]\right) + e_{it}$$

Assuming Ψ is smooth, this is a *generalized additive model* (see, e.g., Hastie & Tibshirani, 1986) or *partially linear model* (see, e.g., Robinson (1988))

Estimation

Under a more general law of motion, Cobb-Douglas production implies

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]\right) + e_{it}$$

Assuming Ψ is smooth, this is a *generalized additive model* (see, e.g., Hastie & Tibshirani, 1986) or *partially linear model* (see, e.g., Robinson (1988))

Problem 1: we require Ψ to be monotonic

- Impose constraints on the 1st derivatives of the smooth function that comprises Ψ (Pya & Wood, 2015)

Estimation

Under a more general law of motion, Cobb-Douglas production implies

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \Psi\left(\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]\right) + e_{it}$$

Assuming Ψ is smooth, this is a *generalized additive model* (see, e.g., Hastie & Tibshirani, 1986) or *partially linear model* (see, e.g., Robinson (1988))

Problem 1: we require Ψ to be monotonic

- Impose constraints on the 1st derivatives of the smooth function that comprises Ψ (Pya & Wood, 2015)

Problem 2: Ψ's argument is a function of the linear parameters

- Use an iterative ‘backfitting’ algorithm (Ichimura & Todd, 2007)

Estimation Protocol

Adapting the Friedman & Stuetzle (1981) algorithm to our setting:

1. Initialize the parameter vector at $\hat{\theta}_0 = (\hat{\beta}_{00}, \hat{\beta}_{k0}, \hat{\beta}_{l0})$
2. For iteration j, calculate $Z_{ij} = \mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \hat{\beta}_{0j-1} - \hat{\beta}_{kj-1}k_{it+1} - \hat{\beta}_{lj-1}\mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$
3. Fit the model $y_{it} = \beta_0 - \beta_k k_{it} - \beta_l l_{it} + \Psi(Z_{ij}) + \epsilon_{it} + \upsilon_{it}$ using the shape constrained estimator of Pya & Wood (2015) to obtain $\hat{\theta}_j$
4. Calculate the Euclidean distance between $\hat{\theta}_j$ and $\hat{\theta}_{j-1}$. If the distance is below some tolerance level, stop and treat $\hat{\theta}_j$ as the model's parameter estimates. If not then set $j \leftarrow j + 1$ and repeat from step 2

- For the remainder of these slides, this algorithm is referred to as NPR
- Convergence proof of NPR in Appendix B (draws on Dominitz & Sherman, 2005)

Some Extensions

- **Measurement Error** in expectations data (cf. de Loecker & Collard-Wexler, 2021, on capital)
  - Extend NPR using Evdokimov & Zeleneev (2025)
  - “EZ-NPR” performs well in Monte Carlos and our application
- **Biased Expectations** - see Appendix D
  - OK if output expectation biases reflected in input biases
  - Productivity expectations bias can be handled by extensions to NPR with panel of expectations
  - But MES data shows little systematic bias (Tab 9) & expectations panel limited
- **Different production function specifications** - e.g., Translog; Value Added instead of gross output.

Outline

Monte Carlo Setup

- We follow ACF (2015) Monte Carlo setup: Details
  - y a Leontief composite of m and a 'value added' function of l and k.
  - ω follows an AR(1) process.
  - Investment subject to a firm-specific convex adjustment cost.
  - Firms face a common, time-invariant wage cost.
  - Optimization error in labor, l (as in ACF) but also in materials, m & investment, i.
- Firms' optimal decisions and expectations have an analytical solution.
- Simulate 1,000 firms over 100 periods, use data from last 10.
- Compare OLS\LP\ACF\NPR across various DGPs.

Monte Carlo Setup

- We follow ACF (2015) Monte Carlo setup: Details
  - y a Leontief composite of m and a 'value added' function of l and k.
  - ω follows an AR(1) process.
  - Investment subject to a firm-specific convex adjustment cost.
  - Firms face a common, time-invariant wage cost.
  - Optimization error in labor, l (as in ACF) but also in materials, m & investment, i.
- Firms' optimal decisions and expectations have an analytical solution.
- Simulate 1,000 firms over 100 periods, use data from last 10.
- Compare OLS\LP\ACF\NPR across various DGPs.

Optimization error in labor choice l (Table 1A in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
ACF $\|(\beta_l, \beta_k \in (0,1))$	0.600	0.600	0.009	0.000	0.401	0.401	0.016	0.000	478

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in labor choice l (Table 1A in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
ACF $\lvert(\beta_l, \beta_k \in (0,1))$	0.600	0.600	0.009	0.000	0.401	0.401	0.016	0.000	478

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in labor choice l (Table 1A in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
ACF $\|(\beta_l, \beta_k \in (0,1))$	0.600	0.600	0.009	0.000	0.401	0.401	0.016	0.000	478

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in labor choice l (Table 1A in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
ACF $\|(\beta_l, \beta_k \in (0,1))$	0.600	0.600	0.009	0.000	0.401	0.401	0.016	0.000	478

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in labor choice l (Table 1A in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.003	0.000	0.400	0.400	0.005	0.000	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.098	0.098	0.004	0.091	500
OP	0.840	0.840	0.004	0.057	0.161	0.161	0.008	0.057	500
LP	0.600	0.600	0.003	0.000	0.400	0.400	0.014	0.000	500
ACF	0.823	0.601	1.043	1.135	0.161	0.401	1.125	1.320	500
ACF $\|(\beta_l, \beta_k \in (0,1))$	0.600	0.600	0.009	0.000	0.401	0.401	0.016	0.000	478

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in all inputs (l, m, i) (Table 1D in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.006	0.000	0.399	0.400	0.034	0.001	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.097	0.097	0.004	0.092	500
OP	0.897	0.897	0.003	0.088	0.104	0.105	0.011	0.088	500
LP	0.304	0.304	0.006	0.088	0.763	0.763	0.020	0.132	500
ACF	0.383	0.347	0.401	0.207	0.664	0.701	0.441	0.264	500
ACF $\|(\beta_l, \beta_k \in (0, 1))$	0.347	0.347	0.028	0.065	0.704	0.701	0.035	0.093	496

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Optimization error in all inputs (l, m, i) (Table 1D in paper)

	$\beta_l = 0.6$				$\beta_k = 0.4$				
	Mean	**Median**	**S.D.**	**MSE**	**Mean**	**Median**	**S.D.**	**MSE**	**N runs**
NPR	0.600	0.600	0.006	0.000	0.399	0.400	0.034	0.001	500
OLS LEVELS	0.919	0.919	0.002	0.102	0.097	0.097	0.004	0.092	500
OP	0.897	0.897	0.003	0.088	0.104	0.105	0.011	0.088	500
LP	0.304	0.304	0.006	0.088	0.763	0.763	0.020	0.132	500
ACF	0.383	0.347	0.401	0.207	0.664	0.701	0.441	0.264	500
ACF $\|(\beta_l, \beta_k \in (0,1))$	0.347	0.347	0.028	0.065	0.704	0.701	0.035	0.093	496

Number of replications given in the 'N runs' column. ACF results from initialization at $\beta_{l0} = 0.5$ and $\beta_{k0} = 0.5$.

Monte Carlo summary

1. NPR robust to optimization errors
    - (Outperforms OLS\LP\ACF as optimization error $\sigma \uparrow$)
2. When optimization error on labor only, NPR, LP and ACF do well – though ACF is at times numerically unstable, see footnote 16 in ACF.
3. When optimization errors affect multiple inputs, NPR remains adequate but other estimators deteriorate as they rely on input choices to proxy for ω.

Outline

Motivation

Methodology: Model, Identification and Estimation

Monte Carlo Simulation

Application

Discussion

Production function estimates (Table 8): Restaurants

Table: Restaurants

	NPR	OLS LVLS	OLS FD	OLS FE	OP	LP	**ACF**
β_l	0.82	0.81	0.63	0.68	0.69	0.69	1.06
	(0.07)	(0.06)	(0.11)	(0.10)	(0.05)	(0.05)	(0.15)
β_k	0.26	0.21	-0.01	0.02	0.09	0.25	0.05
	(0.07)	(0.06)	(0.06)	(0.05)	(0.09)	(0.14)	(0.14)
$\beta_l + \beta_k$	1.08	1.01	0.62	0.70	0.78	0.94	1.10
CRS	0.00	0.33	0.00	0.00	0.03	0.71	0.18
N firms	430	430	392	389	430	430	430

Restaurants have big optimization errors in materials

Production function estimates (Table 8): Electronics Manufacturing

Table: Electronics

	NPR	OLS LVLS	OLS FD	OLS FE	OP	LP	ACF
β_l	0.90	0.86	0.45	0.47	0.60	0.62	0.79
	(0.14)	(0.05)	(0.12)	(0.10)	(0.05)	(0.07)	(0.13)
β_k	0.21	0.23	0.04	0.04	0.26	0.34	0.23
	(0.12)	(0.04)	(0.03)	(0.03)	(0.15)	(0.08)	(0.09)
$\beta_l + \beta_k$	1.11	1.09	0.49	0.51	0.86	0.96	1.02
CRS	0.00	0.00	0.00	0.00	0.36	0.66	0.88
N firms	422	422	411	400	422	422	422

Production function estimates (Table 8): Retail

Table: Retail

	NPR	OLS LVL	OLS FD	OLS FE	OP	LP	ACF
β_l	0.80***	0.75***	0.46***	0.52***	0.63***	0.55***	0.66***
	(0.11)	(0.03)	(0.09)	(0.07)	(0.02)	(0.04)	(0.11)
β_k	0.16***	0.25***	-0.02	-0.02	0.01	0.16***	0.17*
	(0.05)	(0.03)	(0.02)	(0.02)	(0.08)	(0.05)	(0.09)
$\beta_l + \beta_k$	0.96	1.00	0.43	0.49	0.64	0.71	0.83
CRS	0.00	0.80	0.00	0.00	0.00	0.00	0.15
N firms	1853	1853	1807	1763	1853	1853	1853

Stronger Dynamic Moments using NPR

Table: 10: Initial TFP and future firm performance

	NPR	ACF	OLS	Mean	N
		Electronics			
Exit	-0.016***	-0.017***	-0.016***	0.01	5362
	(0.004)	(0.004)	(0.004)		
$l_{t+2} - l_{t+1}$	0.010**	0.008*	0.009**	0.01	2201
Jobs growth (1-yr ahead)	(0.005)	(0.005)	(0.005)		
$l_{t+5} - l_{t+1}$	0.044**	0.03	0.041**	0.025	1398
Jobs growth (4-yr ahead)	(0.021)	(0.020)	(0.020)		
		Retail			
Exit	-0.014***	-0.014***	-0.014***	0.024	57630
	(0.001)	(0.001)	(0.001)		
$l_{t+2} - l_{t+1}$	0.003***	0.003***	0.003***	0.026	13485
Jobs growth (1-yr ahead)	(0.001)	(0.001)	(0.001)		
$l_{t+5} - l_{t+1}$	0.007*	0.005	0.008**	0.095	8331
Jobs growth (4-yr ahead)	(0.004)	(0.004)	(0.004)		

Dynamic Moments: Associations of growth and TFP

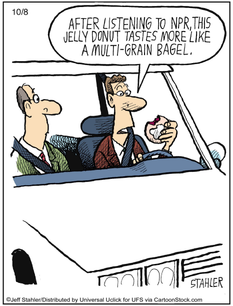
Table: Initial TFP and future firm performance - Restaurants

	NPR	ACF	OLS	Mean	N
	Restaurants				
Exit	-0.007***	-0.003	-0.011***	0.038	12513
	(0.002)	(0.002)	(0.002)		
$l_{t+2} - l_{t+1}$	0.021***	0.012*	0.016**	0.054	2643
Jobs growth (1-yr ahead)	(0.007)	(0.007)	(0.007)		
$l_{t+5} - l_{t+1}$	0.036	0.004	0.005	0.225	1581
Jobs growth (4-yr ahead)	(0.031)	(0.028)	(0.030)		

Outline

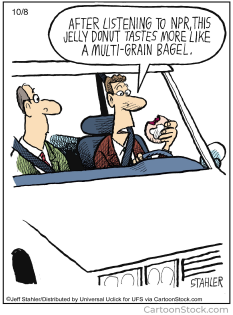
Conclusions

- Subjective expectations data offer a new way of production function estimation: NPR
  - Relax assumptions on firms choosing factor inputs
  - Allows estimation in cross section
- Performs well both in Monte Carlo and in application to UK MES data
- Improved dynamic moments
- Developing downloadable material for new surveys and estimation package



Conclusions

- Subjective expectations data offer a new way of production function estimation: NPR
  - Relax assumptions on firms choosing factor inputs
  - Allows estimation in cross section
- Performs well both in Monte Carlo and in application to UK MES data
- Improved dynamic moments
- Developing downloadable material for new surveys and estimation package




10/8
AFTER LISTENING TO NPR, THIS JELLY DONUT TASTES MORE LIKE A MULTI-GRAIN BAGEL.
STAHLER

Appendix

- Setting $\beta_0 = 0$ for simplicity,

$$y_{it} = \mathbb{E}(y_{it+1}|\Omega_{it})/\rho + \beta_K k_{it} - (\beta_K/\rho)k_{it+1} + \beta_L l_{it} - (\beta_L/\rho)\mathbb{E}(l_{it+1}|\Omega_{it}) + \epsilon_{it}.$$

  Without data on expectations but under rational expectations one could use instead: $y_{it+1} = \mathbb{E}(y_{it+1}|\Omega_{it}) + \zeta^y_{it+1}$ and $l_{it+1} = \mathbb{E}(l_{it+1}|\Omega_{it}) + \zeta^l_{it+1}$. Then

$$y_{it} = (1/\rho)y_{it+1} + \beta_K k_{it} - (\beta_K/\rho)k_{it+1} + \beta_L l_{it} - (\beta_L/\rho)l_{it+1} + e_{it},$$

  where $e_{it} = \epsilon_{it} - (1/\rho)\zeta^y_{it+1} + (1/\rho)\beta_L\zeta^l_{it+1}$. (Both y_{it+1} and l_{it+1} are endogenous regressors.)

- Blundell and Bond (2000) quasi-difference the production function to obtain:

$$y_{it+1} = \rho y_{it} + \beta_K k_{it+1} - \rho\beta_K k_{it} + \beta_L l_{it+1} - \rho\beta_L l_{it} + u_{it+1},$$

  where $u_{it+1} = \xi_{it+1} + \rho\epsilon_{it+1} - \rho\epsilon_{it}$. Equivalently,

$$y_{it} = (1/\rho)y_{it+1} + \beta_K k_{it} - (\beta_K/\rho)k_{it+1} + \beta_L l_{it} - (\beta_L/\rho)l_{it+1} + v_{it+1},$$

  where $v_{it+1} = -(1/\rho)u_{it+1}$. (Both y_{it+1} and l_{it+1} are endogenous regressors.)

Back

Monte Carlo setup

- Following ACF
  - $Y = \min\{K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}, M_{it}\} e^{\epsilon_{it}}$ $(\beta_l = 0.6; \beta_k = 0.4; \epsilon_{it} \sim (0, 0.1))$
  - $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ ($\rho = 0.7$; std dev(ω_{it}) = 0.3)
  - $c_i(I_{it}) = \frac{\phi_i}{2} I_{it}^2$ $(1/\phi_i \sim \ln \mathcal{N}(0, 0.6))$
  - optimization error: $X = X_{it}^* e^{\xi_{it}^x}$ for $X = L, M, I$ and $\xi_{it}^x \sim \mathcal{N}(0, 0.37)$.
- Set $K_{i0} = 0$, simulate the model for 100 periods, use data from the last 10

Back

Monte Carlo setup

- Following ACF
  - $Y = \min\{K_{it}^{\beta_k} L_{it}^{\beta_l} e^{\omega_{it}}, M_{it}\} e^{\epsilon_{it}}$ $\quad (\beta_l = 0.6; \beta_k = 0.4; \epsilon_{it} \sim (0, 0.1))$
  - $\omega_{it} = \rho\omega_{it-1} + \xi_{it}$ $\quad (\rho = 0.7$; std dev$(\omega_{it}) = 0.3)$
  - $c_i(I_{it}) = \frac{\phi_i}{2} I_{it}^2$ $\quad (1/\phi_i \sim \ln \mathcal{N}(0, 0.6))$
  - optimization error: $X = X_{it}^* e^{\xi_{it}^x}$ for $X = L, M, I$ and $\xi_{it}^x \sim \mathcal{N}(0, 0.37)$.
- Set $K_{i0} = 0$, simulate the model for 100 periods, use data from the last 10

Back

Production function estimates: Electronics Manufacturing

Table: Sample: Electronics

	NPR	OLS	OLS FD	OLS FE	OP	LP	ACF
β_l	0.90	0.86	0.45	0.47	0.60	0.62	0.79
	(0.14)	(0.05)	(0.12)	(0.10)	(0.05)	(0.07)	(0.13)
β_k	0.21	0.23	0.04	0.04	0.26	0.34	0.23
	(0.12)	(0.04)	(0.03)	(0.03)	(0.15)	(0.08)	(0.09)
$\beta_l + \beta_k$	1.11	1.09	0.49	0.51	0.86	0.96	1.02
CRS	0.00	0.00	0.00	0.00	0.36	0.66	0.88
N obs.	472	917	458	895	917	917	917
N firms	422	422	411	400	422	422	422

Back

Some Expectations Formation Processes (Pesaran-Weale, 2006)

- Rational Expectations
  - $\mathbb{E}_{it}[\omega_{it}|\Omega_{it-1}] = \mathbb{E}[\omega_{it}|\Omega_{it-1}] = 0$
  - $\mathbb{E}_{it}[\xi_{it+1}|\Omega_{it}] = \mathbb{E}[\xi_{it+1}|\Omega_{it}] = 0$
- Static Expectations (Keynes, 1936)
  - $\mathbb{E}_{it}[\mathbf{x}_{it+1}|\Omega_{it}] = \mathbf{x}_{it}$
  - $\mathbb{E}_{it}[\omega_{it+1}|\Omega_{it}] = \tilde{g}(\omega_{it})$ where $\tilde{g}(\cdot)$ different from $g(\cdot)$.
- "Return to normality" models
  - $\mathbb{E}_{it}[\omega_{it+1}|\Omega_{it}] = (1-\lambda)\omega_{it} + \lambda\omega_i^*$
  - Include fixed effect (either for firm or industry)
- Adaptive Expectations
  - $\mathbb{E}_{it}[\mathbf{x}_{it+1}|\Omega_{it}] = \mathbb{E}_{it-1}[\mathbf{x}_{it}|\Omega_{it-1}] + \Gamma_i(\mathbf{x}_{it} - \mathbb{E}_{it-1}[\mathbf{x}_{it}|\Omega_{it-1}])$
  - Koyck model.
  - Have an extra term in $\Psi(.)$:(1-γ)$\{\mathbb{E}_{it-1}[y_{it}|\Omega_{it-1}] - \int f(k_{it}, l_{it}; \beta)dF_{it-1}(l_{it})\}$
  - Need an extra year of expectations data

Back

Identification

Notice that:

$$\mathbb{E}[y_{it}|z_{it}] = \beta^\top \mathbb{E}[x_{it}|z_{it}] + h(z_{it}) \Rightarrow y_{it} - \mathbb{E}[y_{it}|z_{it}] = \beta^\top (x_{it} - \mathbb{E}[x_{it}|z_{it}]) + \epsilon_{it}$$

$\Rightarrow \beta$ is identified as long as $\mathbb{E}\left[(x_{it} - \mathbb{E}[x_{it}|z_{it}])(x_{it} - \mathbb{E}[x_{it}|z_{it}])^\top\right]$ is non-singular.

[This condition] prevents any element of X from being a.s. perfectly predictable by Z in the least squares sense. (...) Notice that (nonlinear) functional relations among X elements are not ruled out. Notice also that identification may be possible even if X uniquely defines Z, when the converse is not true. Robinson (ECTA, 1988, p.940)

Once we have β the non-parametric regression of $y_{it} - \beta_k k_{it} - \beta_l l_{it}$ on $\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$ identifies h (and, consequently, Ψ and g).

Identification

Notice that:

$$\mathbb{E}[y_{it}|z_{it}] = \beta^\top \mathbb{E}[x_{it}|z_{it}] + h(z_{it}) \Rightarrow y_{it} - \mathbb{E}[y_{it}|z_{it}] = \beta^\top (x_{it} - \mathbb{E}[x_{it}|z_{it}]) + \epsilon_{it}$$

$\Rightarrow \beta$ is identified as long as $\mathbb{E}\left[(x_{it} - \mathbb{E}[x_{it}|z_{it}])(x_{it} - \mathbb{E}[x_{it}|z_{it}])^\top\right]$ is non-singular.

[This condition] prevents any element of X from being a.s. perfectly predictable by Z in the least squares sense. (...) Notice that (nonlinear) functional relations among X elements are not ruled out. Notice also that identification may be possible even if X uniquely defines Z, when the converse is not true. Robinson (ECTA, 1988, p.940)

Once we have β the non-parametric regression of $y_{it} - \beta_k k_{it} - \beta_l l_{it}$ on $\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$ identifies h (and, consequently, Ψ and g).

Identification

Notice that:

$$\mathbb{E}[y_{it}|z_{it}] = \beta^\top \mathbb{E}[x_{it}|z_{it}] + h(z_{it}) \Rightarrow y_{it} - \mathbb{E}[y_{it}|z_{it}] = \beta^\top (x_{it} - \mathbb{E}[x_{it}|z_{it}]) + \epsilon_{it}$$

$\Rightarrow \beta$ is identified as long as $\mathbb{E}\left[(x_{it} - \mathbb{E}[x_{it}|z_{it}])(x_{it} - \mathbb{E}[x_{it}|z_{it}])^\top\right]$ is non-singular.

> *[This condition] prevents any element of X from being a.s. perfectly predictable by Z in the least squares sense. (...) Notice that (nonlinear) functional relations among X elements are not ruled out. Notice also that identification may be possible even if X uniquely defines Z, when the converse is not true.* Robinson (ECTA, 1988, p.940)

Once we have β the non-parametric regression of $y_{it} - \beta_k k_{it} - \beta_l l_{it}$ on $\mathbb{E}_{it}[y_{it+1}|\Omega_{it}] - \beta_k k_{it+1} - \beta_l \mathbb{E}_{it}[l_{it+1}|\Omega_{it}]$ identifies h (and, consequently, Ψ and g).

Identification

This holds when $f(\cdot)$ is linear in parameters (e.g., Cobb-Douglas, translog).

More generally (Theorem 1 in paper),

Assume that $g(\cdot)$ is strictly monotonic and $\mathbb{E}[\epsilon_{it}|\Omega_{it}] = 0$. Let $x_{it} = (k_{it}, l_{it})$ and $z_{it} = (\mathbb{E}_{it}[y_{it+1}|\Omega_{it}], k_{it+1}, F_{it}(\cdot))$ and denote by $\theta_0 = (\beta_0, g_0)$ the data generating parameters. Then, if

$$f(x_{it}; \beta_0) - \mathbb{E}\big[f(x_{it}; \beta_0)|z_{it}\big] \neq f(x_{it}; \beta) - \mathbb{E}\big[f(x_{it}; \beta)|z_{it}\big]$$

with positive probability for any $\beta \neq \beta_0$, the parameter vector $\theta_0 = (\beta_0, g_0)$ is identified.

Back

Predictions and Outcomes

Table: 7: Log Deviation Between Expected Levels and Outcomes

	Mean	S.D.	p50	N obs.	N firms
	Electronics				
Turnover	-0.01	0.29	-0.04	221	205
Employment	0.02	0.18	0.02	221	205
	Retail				
Turnover	-0.05	0.25	-0.03	733	659
Employment	0.00	0.21	0.01	735	661
	Restaurants				
Turnover	-0.18	0.48	-0.13	170	150
Employment	-0.04	0.44	0.03	170	150

Back

Restaurants: more optimization errors for intermediates?

Table: MES vs. ABS: 2017 and 2020

	Mean	S.D.	p50	N obs.	N firms	N p50 obs.	N p50 firms
	MES (2017 & 2020)						
Turnover	0.06	0.37	0.00	8230	7373	50	49
Employment	-0.01	0.26	0.00	8266	7404	50	50
Capex	0.15	1.74	0.00	7501	6757	50	50
Intermediates	0.09	1.43	0.11	8157	7312	50	50
	MES Restaurants (2017 & 2020)						
Turnover	-0.00	0.34	-0.00	235	209	50	48
Employment	-0.03	0.38	0.03	236	209	50	49
Capex	0.62	2.06	0.07	205	180	50	47
Intermediates	0.19	1.03	**0.30**	236	209	50	48
	MES Electronics (2017 & 2020)						
Turnover	0.00	0.17	-0.00	284	252	50	48
Employment	0.02	0.17	0.00	284	252	50	49
Capex	-0.04	1.58	0.01	260	233	50	48
Intermediates	-0.15	1.41	**0.02**	282	250	50	49

Back

References I